A Dual Gauge Model with Confinement

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Abstract

We reformulate the dual gauge model of the long-distance Yang-Mills theory in terms of the two-point Wightman functions formalism. In the flux-tube scheme of Abelian dominance and monopole condensation, the analytic expressions of both monopole- and dual gauge boson-fields propagators are obtained. Formally, new features of higher order quadratic equations for a monopole field are given. Finally, we show how the rising-type potentials in the static system of color charges are naturally derived.

PACS 12.38.Aw, 11.15.Kc, 12.38Aw, 12.38Lg, 12.39Mk, 12.39Pn

1 Introduction

There is a general statement that the color confinement is supported by the idea that the vacuum of quantum Yang-Mills (Y-M) theory is realized

by a condensate of monopole-antimonople pairs [1]. In such a vacuum the interacting field between two colored sources located in $\vec{x}_1$ and $\vec{x}_2$ is squeezed into a tube whose energy $E_{tube} \sim |\vec{x}_1 - \vec{x}_2|$. This is a complete dual analogy to the magnetic monopole confinement in the type II superconductor. Since there is no monopoles as classical solutions with finite energy in a pure Y-M theory it has been suggested by 't Hooft [2] to go into the Abelian projection where the gauge group SU(2) is broken by a suitable gauge condition to its (may be maximal) Abelian subgroup U(1). It is proposed that the interplay between a quark-antiquark pair is analagous to the interaction between a monopole-antimonopole pair in a superconductor.

In fact, the topology of Y-M SU(N) manifold and that of its Abelian subgroup $[U(1)]^{N-1}$ are different, and since any such gauge is singular, one might introduce the string by performing the singular gauge transformation with an Abelian gauge field A_μ [3]

$$A_\mu(x) \to A_\mu(x) + \frac{g}{4\pi}\,\partial_\mu \Omega(x) \ , \tag{1.1}$$

where $\Omega(x)$ is the solid angle subtended by the closed space-like curve described by the string at any point $x = (x^0, x^1)$, and $g = 2\pi/e$ is responsible for the magnetic flux inside the string, e being the Y-M coupling constant. Here, we choose a single string in the two-dimensional (2d) world sheet $y_x(\tau, \sigma)$, for simplicity. Obviously, the Abelian field-strength tensor $F^A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ transforms as

$$F^A_{\mu\nu}(x) \to F^A_{\mu\nu}(x) + \tilde{G}_{\mu\nu}(x) \ ,$$

where a new term

$$\tilde{G}_{\mu\nu}(x) = \frac{g}{4\pi}\,[\partial_\mu, \partial_\nu]\,\Omega(x) \ ,$$

is valid on the world sheet only [4]

$$\tilde{G}_{\mu\nu}(x) = \frac{g}{2}\,\epsilon_{\mu\nu\alpha\beta} \int\!\!\int d\sigma\, d\tau\, \frac{\partial(y^\alpha, y^\beta)}{\partial(\sigma, \tau)}\ \delta_4[x - y(\sigma, \tau)] \ .$$

Formally, a gauge group element, which transforms a generic SU(N) connection onto the gauge fixing surface in the space of connections, is not regular everywhere in spacetime. The projected (or transformed) connections contain topological singularities (or defects). Such a singular transformation (1.1) may form the worldline(s) of magnetic monopoles. Hence, this singularity leads to the monopole current J^{mon}_μ. This is a natural way of the

transformation from the Y-M theory to a model dealing with Abelian fields. A dual string is nothing but a formal idealization of a magnetic flux tube in the equilibrium against the pressure of surrounding superfluid (Higgs-like field) which it displaces [5,6].

Recent lattice results [7] give the promised picture that the monopole degrees of freedom can indeed form a condensate responsible for the confinement. The expression for the static heavy quark potential, using an effective dual Ginzburg-Landau model [8], has been presented in [9]. In the paper [10], an analytic approximation to the dual field propagator without sources and in the presence of quark sources, and an expression for the static quark-antiquark potential were established.

The aim of this paper is to consider the model in 4d based on the dual description of a long-distance Y-M (LDY-M) theory which shows some kind of confinement. We study the model of Lagrangian where the fundamental variables are an octet of dual potentials coupled minimally to three octets of monopole (Higgs) fields. The dual gauge model is studied at the lowest order of the perturbative series using the canonical quantization. The basic manifestation of the model is that it generates the equations of motion where one of them for the scalar Higgs field looks like as a dipole-like field equation. The monopole fields obeying such an equation are classified by their two-point Wightman functions (TPWF). In the classical level there is some intersection with the Froissart model [11] containing the scalar field satisfying the equation of the fourth order. In the scheme presented in this work, the flux distribution in the tubes formed between two heavy color charges is understood via the following statement: the Abelian monopoles are excluded from the string region while the Abelian electric flux is squeezed into the string region.

In Sec. 2, we introduce the essence of the dual gauge Higgs model and the classical solutions. In our model there are the dual gauge field $\hat{C}^a_\mu(x)$ and the monopole field $\hat{B}^a_i(x)$ ($i = 1, ..., N_c(N_c - 1)/2$; a=1,...,8 is a color index) which are relevant modes for infrared behaviour. The local coupling of the $\hat{B}_i$-field to the $\hat{C}_\mu$-field provides the mass of the dual field and, hence, a dual Meissner effect. Although $\hat{C}_\mu(x)$ is invariant under the local transformation of $U(1)^{N_c-1} \subset SU(N_c)$, $\hat{C}_\mu = \vec{C}_\mu \cdot \vec{H}$ is an $SU(N_c)$-gauge dependent object and does not appear in the real world alone (N_c is the number of colors and $\vec{H}$ stands for the Cartan superalgebra). The commutation relations, TPWF

and Green's functions as well-defined distributions in the space $S(\Re^d)$ of complex Schwartz test functions on $\Re^d$ will be defined in Sec. 3. In Sec. 4, we study the monopole- and dual gauge- field propagations. In Sec. 5, we obtain the asymptotic transverse behaviour of both the dual gauge field and the color-electric field. The analytic expression for the static potential is obtained in Sec. 6. Sec. 7 contains the discussion and conclusions.

2 A dual Higgs gauge model and classical solutions

The dual description of the LDY-M theory is simply understood by switching on the dual gauge field $\hat{C}_\mu(x)$ and the three scalar octets $\hat{B}_i(x)$ (necessary to give mass to all C^a_μ and carrying color magnetic charge) in the Lagrangian density (LD) L [12]

$$L = 2\,Tr\left[-\frac{1}{4}\hat{F}^{\mu\nu}\hat{F}_{\mu\nu} + \frac{1}{2}\left(D_\mu \hat{B}_i\right)^2\right] - W\left(\hat{B}_i\right)\ , \tag{2.1}$$

where

$$\hat{F}_{\mu\nu} = \partial_\mu \hat{C}_\nu - \partial_\nu \hat{C}_\mu - ig[\hat{C}_\mu, \hat{C}_\nu]\ ,$$
$$D_\mu \hat{B}_i = \partial_\mu \hat{B}_i - ig\,[\hat{C}_\mu, \hat{B}_i]\ ,$$

$\hat{C}_\mu$ and $\hat{B}_i$ are the SU(3) matrices, g is the gauge coupling constant of the dual theory. The Higgs fields develop their vacuum expectation values (v.e.v.) $\hat{B}_{0_i}$ and the Higgs potential $W(\hat{B}_i)$ has a minimum at $\hat{B}_{0_i}$. The v.e.v. $\hat{B}_{0_i}$ produce a color monopole generating current confining the electric color flux. It is known, the LD (2.1) can generate classical equations of motion carrying a unit of the z_3 flux confined in a narrow tube along the z-axis (corresponding to quark sources at $z = \pm\infty$). This is a dual analogy to the Abrikosov [13] magnetic vortex solution.

As the next step we introduce the color ansatz

$$\hat{C}_\mu = \sum_a C^a_\mu \frac{1}{2}\lambda_a\ , \tag{2.2}$$

where the vector potential C^a_μ is dual to an ordinary vector potential in the Y-M theory, $(1/2)\lambda^a$ are generators of SU(3). Following paper [12] in the

sense of representing the quark sources by the Dirac string tensor $\tilde{G}_{\mu\nu}(x)$ having the same color structure as in (2.2), one can arrive at a more suitable form of the LD (2.1)

$$L(\tilde{G}_{\mu\nu}) = -\frac{1}{3}G^2_{\mu\nu} + 4|(\partial_\mu - igC_\mu)\phi|^2 + 2(\partial_\mu\phi_3)^2 - W(\phi,\phi_3) \ , \qquad (2.3)$$

where

$$G_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu + \tilde{G}_{\mu\nu} \ ,$$

and $\phi(x)$ and $\phi_3(x)$ denote the complex scalar monopole fields. We choose the color structure for the QCD-monopole field $\hat{B}_i$ (belonging to the fundamental representation of $SU_c(3)$) like in [12], and the effective potential stands

$$W(B,\bar{B},B_3) = \frac{2}{3}\lambda\{11[2(B^2+\bar{B}^2-B_0^2)^2 + (B_3^2-B_0^2)^2]$$
$$+7[2(B^2+\bar{B}^2)+B_3^2-3B_0^2]^2\} \ ,$$

where

$$\phi \equiv \phi_1 = \phi_2 = B_{1,2} - i\bar{B}_{1,2} \ , \quad \phi_3 = B_3 \ ,$$

while λ provides the weak couplings between the scalar fields.

The dual gauge field C_μ satisfies the relation

$$\partial_\mu C_\nu - \partial_\nu C_\mu =^* (\partial_\mu A_\nu - \partial_\nu A_\mu)$$

in the absence of charges, and the duality transformation is realized by interchanging the gluon field $A_\mu(x)$ and $C_\mu(x)$. In the maximally Abelian gauge [14] $A_\mu(x) = A^a_\mu(x)(\tau^a/2)$ behaves as the Abelian gauge field $C_\mu(x) = A^3_\mu(x)(\tau^3/2)$ approximately because the off-diagonal fields are suppressed by the gauge transformation. The LD (2.3) is invariant under the local gauge transformation of the dual gauge field C_μ

$$C_\mu(x) \to C_\mu(x) - \partial_\mu\Lambda(x)$$

and the phase transformation of the QCD-monopole field

$$\phi_{1,2}(x) \to \exp[-i\,g\Lambda(x)]\,\phi_{1,2}(x) \ ,$$

where $\Lambda(x)$ is the real field in $S(\Re^3)$ at any fixed x^0. The local gauge symmetry is spontaneously broken because of $\langle\phi\rangle_0 \neq 0$ in (2.3). The generating

current of (2.3) is nothing but the monopole current confining the electric color flux

$$J_\mu^{mon} = 4\, i\, g[\phi^*(\partial_\mu - i\, g\, C_\mu)\, \phi \,-\, \phi(\partial_\mu + i\, g\, C_\mu)\, \phi^*] \ ,$$

which enters into the equation of motion in the form

$$\partial^\nu\, G_{\mu\nu}(x) = \frac{3}{2}\, J_\mu^{mon}(x) \ .$$

The formal consequence of the J_μ^{mon} conservation, $\partial^\mu J_\mu^{mon} = 0$, means that monopole currents form closed loops.

To find a solution of this model, one can consider the monopole-field as a solution where $\langle B(x)\rangle_0 \neq 0$, $\langle \bar{B}(x)\rangle_0 \neq 0$, $\langle B_3(x)\rangle_0 \neq 0$. We choose

$$B(x) = b(x) \,+\, B_0 \ , \bar{B}(x) = \bar{b}(x) \ , B_3(x) = b_3(x) \,+\, B_0$$

with the boundary conditions at large distances ρ from the center of the flux tube with ϑ as an angle in the cylindrical coordinates [12]

$$\vec{C} \to -\frac{e}{g\,\rho}\vec{e}_\vartheta \ , \phi \to B_0 \exp\left(i\vartheta\right) \ , B_3 \to B_0 \ \ as \ \ \rho \to \infty \ . \tag{2.4}$$

In terms of the fields $b(x), \bar{b}(x), b_3(x)$ and $C_\mu(x)$ the LD (2.3) is divided into two parts

$$L = L_1 \,+\, L_2 \ , \tag{2.5}$$

where L_1 in the lowest order of g and λ and with the minimal weak interaction looks like

$$\begin{aligned} L_1 = &-\frac{1}{3}G_{\mu\nu}^2 \,+\, 4\left[(\partial_\mu b)^2 + (\partial_\mu \bar{b})^2 + \frac{1}{2}(\partial_\mu b_3)^2\right] \\ &+ m^2\, C_\mu^2 - \frac{4}{3}\mu^2(50b^2 + 18b_3^2) + 8m\partial_\mu \bar{b}\, C_\mu \ . \end{aligned} \tag{2.6}$$

Here, $m \equiv gB_0$ and $\mu \equiv \sqrt{2\lambda}B_0$ are masses of the dual gauge field and the monopole field, respectively. The remaining part of (2.5) turns out to be

$$L_2 = 8g\left[(\partial_\mu \bar{b})C_\mu b - (\partial_\mu b)C_\mu \bar{b}\right] + 4g^2(\partial_\mu C_\nu - \partial_\nu C_\mu)^2 \cdot (\bar{b}^2 + b^2 + 2B_0 b)$$

$$-\frac{4}{3}\lambda[25(b^4+\bar{b}^4)+9b_3^4+100B_0 b\cdot(\bar{b}^2+b^2+B_0^2)+28B_0(\bar{b}^2+b^2+bb_3)\cdot b_3$$
$$+36B_0 b_3(b_3^2+B_0^2)+2b^2(25\bar{b}^2+7b_3^2)-2B_0^3(50b+18b_3)+56B_0^2 bb_3+14\bar{b}^2 b_3^2].$$

Let us consider the canonical quantization of (2.6). The equations of motion are

$$(\Delta^2+\mu_1^2)\,b(x)=0\ ;$$

$$\Delta^2\,\bar{b}(x)+m(\partial\cdot C)=0\ ; \tag{2.7}$$

$$(\Delta^2+\mu_2^2)\,b_3(x)=0\ ;$$

$$(\Delta^2+m_1^2)\,C_\mu(x)-\partial_\mu(\partial\cdot C)+12\,m\,\partial_\mu\bar{b}(x)-\partial^\nu\tilde{G}_{\mu\nu}(x)=0\ , \tag{2.8}$$

where $\mu_1^2=(50/3)\,\mu^2, \mu_2^2=12\,\mu^2, m_1^2=3\,m^2$. By taking the divergence of (2.8) one can get

$$\Delta^2\,\bar{b}(x)-\frac{1}{9\,m}\partial^\mu\partial^\nu\tilde{G}_{\mu\nu}(x)=0\ , \tag{2.9}$$

while the formal solution of equation (2.8) looks like

$$C_\mu(x)=\alpha\,\partial^\nu\tilde{G}_{\mu\nu}(x)-\beta\,\partial_\mu\bar{b}(x)\ ,$$

where $\alpha\equiv(3\,m^2)^{-1}$, $\beta\equiv 4/m$. We see that the dual gauge field is defined via the divergence of the scalar field $\bar{b}(x)$ shifted by the divergence of the Dirac string tensor $\tilde{G}_{\mu\nu}(x)$. At the same time, we propose in the standard manner the Dirac string which can be understood as a straight line connecting two objects with opposite charges. For large enough $\vec{x}$, approaching such a string, the monopole field is going to its v.e.v. while $C_\mu(\vec{x}\to\infty)\to 0$. Hence,

$$J_\mu^{mon}(\vec{x}\to\infty)\to 8\,m^2\,C_\mu\ .$$

For a very weak C_μ-field one can say that in the $d=2h$ dimensions

$$\Delta^{2h}\,\bar{b}(x)\simeq 0\ ,\quad h=2,3,...\ , \tag{2.10}$$

but

$$\Delta^2 \,\bar{b}(x) \neq 0 \ .$$

Here, the solutions of equation (2.10) obey locality, Poincare covariance and spectral conditions, and look like the dipole "ghosts" at h=2. In this model, the role of the dipole field at d=4 is held in the d=2 dimensions by the simple pole field, and the analogy of the behaviour between the d=2 and d=4 dimensions can be found at the level of Wightman functions in the free case, at least [15]. This is related to the model proposed in [16] in a study of the Higgs phenomenon, from which the present model is distinguished by the coupling to the dual gauge field $C_{\mu}(x)$, and the gauge field strengh tensor is shifted by the Dirac string tensor $\tilde{G}_{\mu\nu}(x)$. Thus, we obtain that the massless scalar field $\bar{b}(x)$ occurs in the model since the symmetry realizes such a way that the LD (2.3) is invariant under the local gauge transformations above mentioned but $\langle \bar{b}(x) \rangle_0 \neq 0$.

Our aim is to find Green's function of a scalar field $\bar{b}(x)$ obeying Eq. (2.10). The propagator $\tau_h(x)$ is defined via TPWF $W_h(x)$ in the d=2 h-dimensions

$$\tau_h(x) = \langle T \,\bar{b}(x)\,\bar{b}(0)\,\rangle_0 = \theta(x^0)\,W_h(x) + \theta(-x^0)\,W_h(-x) \ , \tag{2.11}$$

where

$$W_h(x) = \langle \,\bar{b}(x)\,\bar{b}(0)\,\rangle_0 \tag{2.12}$$

is the distribution in the Schwartz space $S'(\Re^{2h})$ of temperate distributions on $\Re^{2h}$ and obeys the equation

$$\Delta^{2\,h}\, W_h(x) = 0 \ . \tag{2.13}$$

The general solution of eq. (2.13) should be Lorentz invariant and is given in the form [17,18,16] at h=2

$$W_2(x) = a_1 \,\ln \frac{l^2}{-x_{\mu}^2 + i\,\epsilon\,x^0} + \frac{a_2}{x_{\mu}^2 - i\,\epsilon\,x^0} + a_3 \ , \tag{2.14}$$

where a_i (i=1,2,3) are the coefficients to be defined later, while l is an arbitrary parameter with dimension minus one in mass units. Its origin becomes more transparent from [15, 19]. In fact, the first and the second terms in

(2.14) are related to the scalar dipole field and the scalar pole field, respectively [15]. In the first case, the solution is

$$W_2(x) = -i\, E_2^-(x) = \frac{1}{(4\,\pi)^2}\, \ln \frac{l^2}{-x_\mu^2 + i\,\epsilon\, x^0}\ , \tag{2.15}$$

while for the second term the solution looks like

$$W_2(x) = -i\, D_2^-(x) = \frac{1}{(2\,\pi)^2}\, \frac{1}{-x_\mu^2 + i\,\epsilon\, x^0}\ . \tag{2.16}$$

3 TPWF as classical distributions

Before going into the quantization procedure, let us briefly consider the classical distributions (in the sense of generalized functions [20]) of TPWF and T-ordered TPWF for the $\bar{b}(x)$-field at large x_μ^2. The TPWF (2.12) at h=2 is provided by the distribution $\theta(p^0)\,\delta'(p^2)$ as [21]

$$\begin{aligned} W_2(x) \sim \int d_4p\ \theta(p^0)\,\delta'(p^2)\ \exp(-i\,p\,x) \sim -\ln[-\tilde{\mu}^2\,x^2 + i\,\theta(x^0)] \\ = -\left[\ln|\tilde{\mu}^2\,x^2| + i\,\pi\, sgn(x^0)\,\theta(x^2)\right]\ , \end{aligned} \tag{3.1}$$

where $\tilde{\mu}$ is an arbitrary parameter (the infrared regularization parameter). The distribution $\theta(p^0)\,\delta'(p^2)$ in (3.1) is uniquely defined only on the test functions $f(p) \in S_0(\Re_4)$ ($S_0(\Re_4) = \{f(p) \in S(\Re_4), f(p=0) = 0\}$)

$$\begin{aligned} \int d_4p\ \theta(p^0)\,\delta'(p^2)\, f(p) &= \lim_{\nu^2 \to 0} \frac{\partial}{\partial \nu^2} \int d_4p\ \theta(p^0)\,\delta'(p^2 - \nu^2)\, f(p)\ , \\ &= \int_{\Gamma_0^+} \frac{d_3\vec{p}}{2p^0}\, \frac{1}{2\,(n\,p)} \left[\frac{1}{(n\,p)} - (n\,\partial)\right] f(p) \end{aligned}$$

and for an arbitrary fixed time-like unit vector n_μ in the case we choose $n_\mu = (1, \vec{0})$ from V^+ where

$$V^+ = \left\{x \in \Re\, :\, x^0 > |x| \leq \left[\Sigma_{j=1}^3 (x_j)^2\right]^{1/2}\right\}$$

is an open upper light cone in the M-space. Under the dilatation transformation $x \to \alpha x$ ($\alpha > 0$) the TPWF (3.1) acquires the additional term

$$W_2(x) \to W_2(\alpha x) = W_2(x) - \frac{1}{2\,(2\,\pi)^2}\,\ln\alpha. \tag{3.2}$$

It could be interpreted as a spontaneous symmetry breaking of the dilatation invariance of (2.10). This is an important point in the special role of the field $\bar{b}(x)$.

In general, the $\tau_h(x)$-function (2.11) is a well-defined distribution in $S'(\Re^{2h})$ and obeys the h-ordered quadratic differential equation in the d-dimension of space-time

$$\Delta^{2h}\,{\tau_h}^d(x) = \delta_d(x)\ , \quad h = 2, 3, ...\ ,$$

$$\Delta^2 \equiv \frac{\partial^2}{\partial\,{x^2}_1} + ... + \frac{\partial^2}{\partial\,{x^2}_m} - \frac{\partial^2}{\partial\,{x^2}_{m+1}} - ... - \frac{\partial^2}{\partial\,{x^2}_{m+n}}\ ;$$

in the following cases [20,22]
i) even d and $h \geq d/2$

$$\begin{aligned}{\tau_h}^d(x) = (-1)^{d/2-1}\,\frac{\exp[(\pi/2)\,i\,n]}{4^h\,(h-d/2)!\,(h-1)!}\,\left[\tilde{\mu}^2\,P(x) + i\,\epsilon\right]^{-d/2+h} \\ \cdot\ln\left[\tilde{\mu}^2\,P(x) + i\,\epsilon\right]\ ,\end{aligned} \tag{3.3}$$

where $P(x) = x_1^2 + ... + x_m^2 - x_{m+1}^2 - ... - x_{m+n}^2$;
ii) even d and $h < d/2$

$${\tau_h}^d(x) = (-1)^h\,\frac{\exp[(\pi/2)\,i\,n]}{4^h\,\pi^{d/2}\,(h-1)!}\,\left[\tilde{\mu}^2\,P(x) + i\,\epsilon\right]^{-d/2+h}\,\Gamma\left(\frac{d}{2} - h\right)\ . \tag{3.4}$$

In the case of $d = 2h$-dimensions, expression (3.3) can be more simplified as follows:

$${\tau_h}^{d=2h}(x) = (-1)^{h-1}\,\frac{\exp[(\pi/2)\,i\,n]}{4^h\,(h-1)!}\,\ln\,\left[\tilde{\mu}^2\,P(x) + i\,\epsilon\right]\ .$$

For the following nearly realistic consideration of the confinement occurrence, we are interested in the case i) (3.3) which allows one to study highly singular objects for the confinement-like picture. In view of that, for $h = 2$ in

the M-space following Zwanziger [23] the Fourier inversion $F[{\tau_2}^M(x)]$ of the distribution ${\tau_2}^M(x)$ looks like

$$F\left[\,{\tau_2}^M(x)\right] = \frac{\pi^2}{2}\,\frac{\partial}{\partial\, p^\mu}\,\left[\frac{p^\mu\,\ln(l^2\,p^2 - i\,\epsilon)}{(p^2 + i\,\epsilon)^2}\right] \tag{3.5}$$

and obeying the equation

$$(p^2)^2\,F\left[{\tau_2}^M(x)\right] = 1\ , \quad p^2 = (p^0)^2 - \Sigma_{j=1}^3\,p_j^2\ ,$$

where $l = \exp(\gamma_E - 1/2)\cdot\,(2\,\tilde{\mu})^{-1}$ (γ_E stands for the Euler constant). Note that in terms of weak derivatives the distribution (3.5) can be rewritten as [15]

$$F\left[{\tau_h}^d(x)\right] = \lim_{\kappa^2\to 0}\,\left[\frac{(-1)^{d/2}}{(p^2 - \kappa^2 + i\,\epsilon)^{d/2}} + i\,\pi^{d/2}\,\delta_d(p)\,\ln\left(\frac{\kappa^2}{4\,\tilde{\mu}^2}\right)\right]\ . \tag{3.6}$$

for any h allowed in the dimension d. The remaining case ii) (3.4) leads to the following singular object:

$$F\left[\,{\tau_h}^d(x)\right] = \lim_{\kappa\to 0}\,\tilde{\mu}^{-d}\,\left(-X^2 + i\,\kappa\right)^{-h}\ ,$$

where $\tilde{\mu}^2\,X^2 = p_1^2 + ... + p_m^2 - p_{m+1}^2 - ... - p_{m+n}^2$. In the space M one has

$$F\left[\,{\tau_h}^M(x)\right] = \lim_{\kappa\to 0}\,\tilde{\mu}^{-4}\,\left(\frac{\tilde{\mu}^2}{-p^2 + i\,\epsilon}\right)^2\ .$$

4 A dual field propagator

For calculation of the coefficients a_1 and a_2 in (2.14) one has to introduce invariant functions $E_2(x) = E_2^-(x) - E_2^-(-x)$ and $D_2(x) = D_2^-(x) - D_2^-(-x)$ [23] (see also formulae (2.15) and (2.16)) which define the commutator written in the general form [16]

$$\left[\,\bar{b}(x), \bar{b}(0)\,\right] = (2\,\pi)^2\,i\,\left[4\,a_1\,E_2(x) + a_2\,D_2(x)\right]\ , \tag{4.1}$$

$$E_2(x) = (8\,\pi)^{-1}\,sgn(x^0)\,\theta(x^2)\ ,$$

$$D_2(x) = (2\,\pi)^{-1}\, sgn(x^0)\, \delta(x^2)\ .$$

In the space $S'(\Re^4)$ on $\Re^4$ the propagator for the $\bar{b}(x)$-field looks like [16]

$$\tau_2(x) = a_1 \left[\ln|\,\tilde{\mu}^2\, x_\mu^2\,| + i\,\pi\,\theta(x_\mu^2) \right] + a_2\, \left[x_\mu^{-2} + i\,\pi\,\delta(x_\mu^2) \right] + a_3\ . \qquad (4.2)$$

The coefficients a_1 and a_2 in (4.2) can be fixed using the canonical commutation relations (CCR)

$$[C_\mu(x), \pi_{C_\nu}(0)]_{|_{x^0=0}} = i\, g_{\mu\nu}\, \delta^3(\vec{x})\ ,$$

$$\left[\bar{b}(x), \pi_{\bar{b}}(0) \right]_{|_{x^0=0}} = i\, \delta^3(\vec{x})\ ,$$

respectively. Here, the conjugate momenta $\pi_{C_\mu}(x)$ and $\pi_{\bar{b}}(x)$ look like

$$\pi_{\bar{b}}(x) = 8\, \left[\partial^0\, \bar{b}(x) + m\, C^0(x) \right]\ ,$$

$$\pi_{C_\mu}(x) = -\frac{4}{3}\, G_{0\mu}(x)\ . \qquad (4.3)$$

The direct calculation leads to (see also Appendix):

$$a_1 = \frac{m^2}{12\,(2\,\pi)^2}\ ,$$

$$a_2 = -\frac{1}{6\,(2\,\pi)^2}\ .$$

Restricting the $W_2(x)$ function (2.14) to only the first term, one can obtain that $\tau_2(x)$ satisfies the equation

$$(\Delta^2)^2\, \tilde{\tau}_2(x) = i\, \delta_4(x)\ , \qquad (4.4)$$

where $\tilde{\tau}_2(x) = 3(2/m)^2\, \tau_2(x)$. The formal Fourier transformation in $S'(\Re_4)$ gives

$$\hat{\tau}_2(p) = weak \lim_{\tilde{\kappa}^2 << 1} \frac{i}{3\,(2\,\pi)^4}\, \left\{ m^2 \left[\frac{1}{(p^2 - \kappa^2 + i\,\epsilon)^2} + i\,\pi^2\, \ln \frac{\kappa^2}{\tilde{\mu}^2}\, \delta_4(p) \right] \right.$$

$$-\frac{1}{2}\,\frac{1}{p^2-\kappa^2+i\,\epsilon}\Bigg\}\,. \quad (4.5)$$

Here, κ is a parameter of representation and not the analogue of the infrared mass, $\tilde{\kappa}^2 \equiv \kappa^2/p^2$. To derive (4.5), we used the well-known mathematical trick with the generalized functions [20,16,15]

$$weak \lim_{\tilde{\kappa}^2<<1} \int d_{2h}p\, \exp(-i\,p\,x)\, \frac{(-1)^h}{(p^2-\kappa^2+i\,\epsilon)^h} =$$

$$= \frac{2\,i}{\Gamma(h)\,(4\,\pi)^h}\{\ln 2 - \gamma_E - \ln(\kappa\sqrt{-x_\mu^2+i\,\epsilon})$$

$$+O[-\kappa^2\,x_\mu^2, -\kappa^2\,x_\mu^2\, \ln(\kappa\,\sqrt{-x_\mu^2+i\,\epsilon})]\}\ .$$

To go into the structure of the dual C_μ-field propagator, we need to define the general form of the commutation relation $[\,C_\mu(x), C_\nu(y)\,]$. To do this procedure let us consider the canonical conjugate pair $\{C_\mu, \pi_{C_\nu}\}$ (for the LD (2.6)) where $\pi_{C_\mu}(x)$ results from (4.3). The consequent CCR looks like

$$\left[\frac{4}{3}C_\mu(x), \partial_\nu C_0(0) - \partial_0 C_\nu(0) - g_{0\nu}(\partial\cdot C(0)) + \Delta_{0\nu}(0)\right]_{|_{x^0=0}} = i g_{\mu\nu}\delta^3(\vec{x}), \quad (4.6)$$

where $\Delta_{\mu\nu}(x) = g_{\mu\nu}\,(\partial\cdot C(x)) - \tilde{G}_{\mu\nu}(x)$ tends to zero as $x\to 0$ and the Dirac string tensor $\tilde{G}_{\mu\nu}(x)$ obeys the equation

$$\left(\Delta^2 + M^2\right)\,\tilde{G}_{\mu\nu}(x) = 0\ , \quad as\ x\to 0.$$

Here, the equations of motion (2.7) and (2.9) were used, and $M = 3m$. Obviously, the following form of the free C_μ-field commutator (see Appendix)

$$[\,C_\mu(x), C_\nu(0)] = i g_{\mu\nu}\left[\,\xi\, m_1^2\, E_2(x) + c\, D_2(x)\right]\ , \quad (4.7)$$

ensures the CCR (4.6) at large x_μ^2. Here, both ξ and c are real arbitrary numbers. The TPWF for $C_\mu(x)$ stands as (see also [15,19])

$$w_{\mu\nu}(x) = \langle C_\mu(x) C_\nu(0)\rangle_0 = i g_{\mu\nu}\left[\xi m_1^2 E_2^-(x) + c D_2^-(x) + c_1 F_2^-(x) + c_2\right], \quad (4.8)$$

where $F_2^-(x) = \langle\,\alpha(x)\,\alpha(0)\,\rangle_0 = i\,x^2$ is TPWF for the harmonic field $\alpha(x)$, c_1 and c_2 are arbitrary real numbers. In fact, the third and the fourth terms

in (4.8) did not appear in (4.7) due to the properties of the $F_2^-(x)$-function, namely, $F_2(x) = F_2^-(x) - F_2^-(-x) = 0$ and the triviality, respectively. One can easily verify that (4.6) and (4.7) can be used to derive the following requirement for ξ

$$\xi = \frac{3}{4} - 4\,c\ . \tag{4.9}$$

The free dual gauge field propagator in $S'(\Re_4)$ in any local covariant gauge can be assumed as

$$\begin{aligned}\hat{\tau}_{\mu\nu}(p) &= \int d^4x\ \exp(i\,p\,x)\,\tau_{\mu\nu}(x) \\ &= i\left[g_{\mu\nu} - \left(1-\frac{1}{\zeta}\right)\frac{p_\mu\,p_\nu}{p^2+i\,\epsilon}\right]\cdot\left[\xi\,m_1^2\,\hat{t}_1(p) + c\,\hat{t}_2(p)\right]\ ,\end{aligned} \tag{4.10}$$

where

$$\tau_{\mu\nu}(x) = \frac{i\,g_{\mu\nu}}{(4\,\pi)^2}\left[\xi\,m_1^2\,\ln(-\tilde{\mu}^2\,x_\mu^2 + i\,\epsilon) + \frac{c}{x_\mu^2 + i\,\epsilon}\right]\ ;$$

$$\hat{t}_1(p) = weak\ \lim_{\tilde{\kappa}^2<<1}\left[\frac{1}{(p^2-\kappa^2+i\,\epsilon)^2} + i\,\pi^2\,\ln\left(\frac{\kappa^2}{\tilde{\mu}^2}\right)\,\delta_4(p)\right]\ ;$$

$$\hat{t}_2(p) = weak\ \lim_{\tilde{\kappa}^2<<1}\frac{1}{2}\,\frac{1}{(p^2-\kappa^2+i\,\epsilon)}\ .$$

The gauge parameter ζ in (4.10) is a real number. According to Eq. (4.4), the following requirement on Green's function $\tau_{\mu\nu}(x)$

$$(\Delta^2)^2\,\tilde{\tau}_{\mu\nu}(x) = i\,\delta_4(x)\ ,$$

leads to that a constant c has to be equal to zero where $\tilde{\tau}_{\mu\nu}(x) = \tau_{\mu\nu}(x)/(\xi\,m_1^2)$.

At the end of this section, let us consider the v.e.v.

$$V_\mu(x) = \left\langle\left[\,J_\mu^{mon}(x), \bar{b}(0)\right]\right\rangle_0\ , \tag{4.11}$$

where the monopole current expressed in terms of the $\bar{b}$-field and $\tilde{G}_{\mu\nu}$ Dirac tensor is

$$J_\mu^{mon}(x) = \frac{2}{3}\left[\frac{3}{m}\,\partial_\mu\Delta^2\,\bar{b}(x) - \alpha\,\Delta^2\,\partial^\nu\tilde{G}_{\mu\nu}(x) - \partial^\nu\tilde{G}_{\mu\nu}(x)\right]\ .$$

Using the commutation relation (4.1) and the well-known relation between the following distributions like $\Delta^2\, sgn(x^0)\,\theta(x^2) = 4\, sgn(x^0)\,\delta(x^2)$, we arrive at the formal expression for $V_\mu(x)$

$$V_\mu(x) = \frac{i\,m}{12\,\pi}\,\partial_\mu\left[sgn(x^0)\,\delta(x^2)\right] - \frac{2}{3}\Big\langle\,(\alpha\,\Delta^2 - 1)\,\left[\partial^\nu \tilde{G}_{\mu\nu}(x), \bar{b}(0)\right]\Big\rangle_0\,. \qquad (4.12)$$

It is easily seen that the requirement of the Goldstone theorem for occurrence of a $\delta(p_\mu^2)$ term in the Fourier transformation of V_μ is satisfied due to the presence of the first term in (4.12) which gives the Fourier transformed term $\sim p_\mu\, sgn(p^0)\,\delta(p^2)$. The second term in (4.12) comes from the dual string effect.

5 The LDY-M solution for a gauge field

In this section, we consider the approximate topological solution for the dual gauge field in the LDY-M theory. The equations of motion are

$$\partial^\nu G_{\mu\nu} = 6\,i\,g[\phi^*(\partial_\mu - i\,g\,C_\mu)\,\phi \;-\; \phi(\partial_\mu + i\,g\,C_\mu)\,\phi^*]\;; \qquad (5.1)$$

$$(\partial_\mu - i\,g\,C_\mu)^2\,\phi = \frac{2}{3}\,\lambda\,(32\,B_0^2 - 25\,|\,\phi\,|^2 - 7\,\phi_3^2\,)\,\phi\;. \qquad (5.2)$$

At large distances Eq. (5.2) transforms into the following one:

$$(\partial_\mu - i\,g\,C_\mu)^2\,\phi = \frac{50}{3}\,\lambda\,(\,B_0^2 - \,|\,\phi\,|^2\,)\,\phi\;.$$

To get the solution, let us do the polar decomposition of the monopole field $\phi(x)$ using new scalar variables $\chi(x)$ and $f(x)$

$$\phi(x) = \frac{1}{\sqrt{2}}\,\exp(i\,f(x))\,[\,\chi(x) + B_0\,]\;.$$

Then, the equation of motion (5.1) transforms into the following one:

$$\partial^\nu G_{\mu\nu} = 6\,g\,(\chi + B_0)^2\,(g\,C_\mu - \partial_\mu\,f)\;. \qquad (5.3)$$

This means that the $\bar{b}(x)$-field is nothing but a mathematical realization of the "massive" phase $B_0 \cdot f(x)$ at large enough distancese:

$$\bar{b}(x) = \frac{B_0}{2}\,S(x)\,f(x) +$$

$$m\int[\,2\,S(x)-1]\,C_\mu(x)\,d\,x^\mu+\frac{1}{12\,m}\int[\,\Delta^2\,C_\mu(x)-\partial^\nu\partial_\mu\,C_\nu(x)]\,d\,x^\mu\ , \quad (5.4)$$

where $S(x)\equiv(1+\chi(x)/B_0)^2$. Now we have to define the flux as

$$\Pi=\int G_{\mu\nu}(x)\,d\,\sigma^{\mu\nu}\ , \quad (5.5)$$

where $\sigma^{\mu\nu}$ is the 2d surface element in the M-space. A substitution of $C_\mu(x)$ from (5.3) into (5.5) gives

$$\Pi=\frac{\alpha}{2}\oint_\Gamma S^{-1}(x)\,\partial^\nu\tilde G_{\mu\nu}(x)\,d\,x^\mu+\int\tilde G_{\mu\nu}(x)\,d\,\sigma^{\mu\nu}+\frac{1}{g}\oint_\Gamma\partial_\mu\,f(x)\,d\,x^\mu\ , \quad (5.6)$$

where Γ means the large closed loop where the current $\partial_\mu C_\nu-\partial_\nu C_\mu$ is canceled. Integrating out over the loop Γ in the third term in (5.6) is nothing but the requirement that the phase $f(x)$ is varied by $2\,\pi\,n$ for any integer number n associated with the topological charge [24] inside the flux tube. One can present the fields $C_\mu(x)$ and $\phi(x)$ in the cylindrical symmetry case using the radial coordinate r (see also [24])

$$\vec C\to\frac{\tilde C(r)}{r}\,\vec e_\theta\ ,\quad \phi\to\phi(r)\ ,$$

and $f=2\,\pi\,n=n\,\theta$ with θ being the azimuth around the z-axis. Thus, the field equation looks like

$$\frac{d^2\,\tilde C(r)}{d\,r^2}-\frac{1}{r}\,\frac{d\,\tilde C(r)}{d\,r}-3\,m^2\,[\,3+2\,S(r)]\,\tilde C(r)+6\,n\,m\,B_0\,S(r)=0\ . \quad (5.7)$$

The following boundary conditions:

$$\tilde C(r)=\frac{4\,n}{7\,g}\ ,\quad \chi(r)=(\sqrt{2}-1)\,B_0\ ,\ as\ r\to\infty\ ;$$

$$\tilde C(r)=0\ ,\quad \chi(r)=0\ ,\ as\ r\to 0$$

are conjugate with those in (2.4). At large enough $r>>\mu^{-1}$ (($\mu=\sqrt{2\,\lambda}\,B_0)^{-1}$ defines the transverse dimension(s) of a monopole field around the tube) Eq. (5.7) transforms into the following one:

$$\frac{d^2\,\tilde C(r)}{d\,r^2}-\frac{1}{r}\,\frac{d\,\tilde C(r)}{d\,r}+3\,g\,(4\,n-7\,g\,\tilde C)\,B_0^2=0$$

with the asymptotic transverse behaviour of its solution

$$\tilde{C}(r) \simeq \frac{4\,n}{7\,g} - \sqrt{\frac{\pi\,m\,r}{2\,k}}\,e^{-k\,m\,r}\,\left(1 + \frac{3}{8\,k\,m\,r}\right)\ ,\quad k \equiv \sqrt{21}\ .$$

The field $\tilde{C}(r)$ grows rapidly when the radial distance from the center of the flux tube $r < r_0 \simeq 3\ fm$ and approaches $4\,n/7\,g$ as soon as $r \geq r_0$. Now it will be very instructive to clarify the singular properties of the field $\bar{b}(x)$ related to the phase $f(x)$ by means of (5.4). Since the phase is provided by the asimuth θ around the z-axis, $\vec{\nabla}\,f = (n/r)\,\vec{e}_\theta$. In the strong limit of the mass m of the C_μ field (as well as the mass of the monopole field) at large distances

$$\vec{\nabla} \times \vec{\nabla}\,\bar{b} = 2\,\pi\,n\,B_0\,\delta(x)\,\delta(y)\,\vec{e}_z\ ,$$

where $\vec{e}_z$ is the unit vector along the axis z and the δ-functions stand for the center of the flux tube [24]. Hence, the real singular character of the field $\bar{b}(x)$ is confirmed by its singular behaviour at the center of the flux tube for nonzero monopole condensate. At the end of this section we give the transverse distribution in r of the color electric field $E_z(r)$ (see also [24])

$$\hat{E}_c = \vec{\nabla} \times \vec{C} = \frac{1}{r}\,\frac{d\tilde{C}(r)}{d\,r}\,\vec{e}_z \equiv E_z(r)\cdot\vec{e}_z$$

which has the following profile in the flux tube at large r

$$E_z(r) = \sqrt{\frac{\pi\,m}{2\,k\,r}}\,\left(k\,m - \frac{1}{2\,r}\right)\,e^{-k\,m\,r}\ .$$

6 Static potential

In this section, we intend to obtain the confinement potential in an analytic form for the system of interacting static test charges of quark and antiquark. Our statement is based on the dual character of the field $C_\mu(x)$ to a gluon field where $C_\mu(x)$ is just the interacting field provided by the monopole field $\bar{b}(x)$ and the divergence of $\tilde{G}_{\mu\nu}(x)$. We have found that $\bar{b}(x)$ plays the role of the dipole-type field at $h = 2$ (see Eq. (2.10)). The mass of the dual field $C_\mu(x)$ is nonzero and equal to $m = g\,B_0$. It is assumed that the mass m results from the Higgs-like mechanism when the dual field $C_\mu(x)$ interacts

with the $\phi(x)$ field, namely an octet of dual potentials C_μ coupled weakly with three octets of scalar fields $\phi_\alpha(x)(\alpha = 1,2,3)$.

According to the distribution (3.6), the first term in the expansion for the static potential

$$P_{stat}(r) = \int d_3\vec{p}\, e^{i\,\vec{p}\,\vec{r}}\, F\{\tau_h{}^d(x)\}_{|_{p^0=0}}$$

in $\Re^3$ is a rising function with $r = |\vec{x}|$ [25,22]

$$P_{stat}(r) \sim \frac{1}{2^{2\,h}\,\pi^{3/2}}\,\frac{1}{(h-1)!}\,\Gamma(3/2-h)\,r^{2\,h-3}\ . \qquad (6.1)$$

It is obvious that at $h \geq 2$ the distribution (6.1) increases with r linearly ($h = 2$) or faster ($h > 2$). Let us represent the distribution r^σ as the Taylor series around some regularization point σ_0

$$r^\sigma = \tilde{\mu}^{-\omega}\,r^{\sigma_0}\,\left[1 + \omega\,\ln(\tilde{\mu}\,r) + \frac{1}{2}\,\omega^2\,\ln^2(\tilde{\mu}\,r) + ...\right]\ , \qquad (6.2)$$

where $\omega = \sigma - \sigma_0$ is an infinitesimal positive interval at $\sigma \neq -d, -d-2, ...$ The potential (6.1) is simplified to [22]

$$P_{stat}(r) \sim \frac{1}{8\,\pi}\,r\,[1 + \ln(\tilde{\mu}\,r) + ...]\ .$$

The Fourier transform $F\{r^\sigma\}$ of (6.2) into $S'(\Re_d)$ for the limit $\omega \to 0$ leads to the following singular distribution in the whole region of the existence of the analytic function r^σ at $\sigma \neq -d, -d-2, ...$:

$$F\{r^\sigma\} = \left(\frac{4\,\pi}{p^2}\right)^{(\sigma+d)/2}\,\pi^{-\sigma/2}\,\frac{\Gamma[(\sigma+d)/2]}{\Gamma(-\sigma/2)}\ .$$

In general, the static potential is defined as

$$P_{stat}(r) = \lim_{T\to\infty}\frac{1}{T}\,A(r)\ , \qquad (6.3)$$

where the action $A(r)$ is given by the colour source-current part of LD

$$L(p) = -\vec{j}^{\,\mu}_{\alpha}(-p)\,\hat{\tau}_{\mu\nu}(p)\,\vec{j}^{\,\nu}_{\alpha}(p)\ .$$

It is known that for such a system of heavy particles the sources are given by a c-number current

$$\vec{j}^{\mu}_{\alpha}(x) = \vec{Q}_{\alpha}\, g^{\mu 0}\, [\delta_3(\vec{x}-\vec{x_1}) - \delta_3(\vec{x}-\vec{x_2})]$$

with $\vec{Q}_{\alpha} = e\,\vec{\rho}_{\alpha}$ being the Abelian color-electric charge of a quark while $\vec{\rho}_{\alpha}$ is the weight vector of the SU(3) algebra: $\rho_1 = (1/2,\, \sqrt{3}/6)$, $\rho_2 = (-1/2,\, \sqrt{3}/6)$, $\rho_3 = (0,\, -1/\sqrt{3})$ [24]. Here, $\vec{x}_1$ and $\vec{x}_2$ are the position vectors of a quark and an antiquark, respectively; the label α=(1,2,3) corresponds to the color electric charge. The calculation of the potential (6.3) is most easily performed by taking into account the Fourier transformed quark current

$$\vec{j}_{\mu\alpha}(p) = 2\,\pi\,\vec{Q}_{\alpha}\, g_{\mu 0}\, \delta(p^0)\, \left(e^{-i\,\vec{p}\,\vec{x}_1} - e^{-i\,\vec{p}\,\vec{x}_2}\right)\ , \tag{6.4}$$

and by making use of the representation in the sense of generalized functions [23]

$$weak \lim_{\tilde{\kappa}^2 << 1} \left[\frac{1}{(p^2-\kappa^2+i\,\epsilon)^2} + i\,\pi^2\, \ln\frac{\kappa^2}{\tilde{\mu}^2}\, \delta_4(p)\right] =$$
$$= \frac{1}{4}\frac{\partial^2}{\partial\, p^2}\, \frac{1}{-p^2-i\,\epsilon}\, \ln\frac{-p^2-i\,\epsilon}{\tilde{\mu}^2} = \frac{1}{2}\, \frac{1}{(p^2+i\,\epsilon)^2}\, \left(5 - 3\, \ln\frac{-p^2-i\,\epsilon}{\tilde{\mu}^2}\right)\ . \tag{6.5}$$

Due to the presence of the $\delta(p^0)$-function (see (6.4) in evaluating of the action $A(r)$, the remaining 3-dimensional integral over d_3p will be easily calculated out using the instructive formula [20,15]

$$\int d_3\vec{p}e^{i\vec{p}\vec{x}} p^m \ln^{\beta} p = \frac{1}{r^{m+3}} \sum_{i=1}^{\beta} \frac{\Gamma(\beta+1)(-1)^i}{\Gamma(\beta-i+1)\Gamma(i+1)} \left(\frac{d}{d\,m}\right)^{\beta-i} H_m \ln^i r, \tag{6.6}$$

where

$$H_m = 2^{m+3}\, \pi^{3/2}\, \frac{\Gamma[(m+3)/2]}{\Gamma(-m/2)}\ ,\quad m \neq -3, -5, \dots\ ,$$

$r \equiv |\vec{x}|$, $p \equiv |\vec{p}|$, $\vec{x} \in \Re^3$, $\vec{p} \in \Re_3$.

As a consequence of the dual field propagator (4.10) the static potential (6.3) at large distances looks like

$$P_{stat}(r) = \frac{\vec{Q}^2}{16\,\pi}\left\{\xi\, m^2\, r[5 + 6(A + \ln\tilde{\mu}\, r)] + O\left(\frac{c}{r}\right)\right\}\ , \tag{6.7}$$

where $A \equiv 1 + \sqrt{\pi} - (1/2 + 2\sqrt{\pi}\,\gamma_E) - (2\sqrt{\pi} + 1)\ \ln 2 < 0$ and the last term in (6.7) is just the positive correction and not the analogue of the Coulomb part of the potential due to the one-gluon exchange. Neglecting the last term in (6.7) and taking into account formula (4.9), one can conclude

$$P_{stat}(r) \simeq \frac{3\,\vec{Q}^2}{64\,\pi}\, m^2\, r(-12.4 + 6\ \ln \tilde{\mu}\, r)\ . \tag{6.8}$$

Hence, the string tension a in $P_{stat}(r) = a\, r$ emerges as

$$a \simeq \frac{\alpha_s}{16}\, m^2\left(-12.4 + 3\ \ln \frac{\tilde{\mu}^2}{m^2}\right), \quad \tilde{\mu} > 9\, m\ , \tag{6.9}$$

where r in the logarithmic function in (6.8) has been changed by the characteristic length $r_c \sim 1/m$ which determines the transverse dimension of the dual field concentration, while $\tilde{\mu}$ is associated with the "coherent length" inverse and the dual field mass m defines the "penetration depth" in the type II superconductor. For a typical value of the electroweak scale $\tilde{\mu} \simeq 250\ GeV$ we get $a \simeq 0.20\ GeV^2$ for the mass of the dual C_μ-field $m = 0.6\ GeV$ and $\alpha_s = e^2/(4\,\pi)$=0.37 obtained from fitting the heavy quark-antiquark pair spectrum [26]. The value of the string tension (6.9) is quite close to a phenomenological one (eg., coming from the Regge slope of the hadrons). Making the formal comparison of the result obtained here in the analytic form let us remind the analogue with the well-known expression of the energy per unit length of the vortex in the type II superconductor [27,9]

$$\epsilon_1 = \frac{{\phi_0}^2\, m_A^2}{32\,\pi^2}\ \ln\left(\frac{m_\phi}{m_A}\right)^2\ , \tag{6.10}$$

where ϕ_0 is the magnetic flux of the vortex, m_A and m_ϕ are penetration depth and the coherent length inverse, respectively. On the other hand, the string tension in Nambu's paper (see the first ref. in [1]) is given by

$$\epsilon_2 = \frac{g_m^2\, m_v^2}{8\,\pi}\ \ln\left(1 + \frac{{m_s}^2}{{m_v}^2}\right)\ , \tag{6.11}$$

with m_s and m_v being the masses of scalar and vector fields and g_m is a magnetic-type charge. It is clear from formula (6.7) that for a sufficiently long string $r >> m^{-1}$ the $\sim r$-behaviour of the static potential is dominant; for a short string $r << m^{-1}$ the singular interaction provided by the second term in (6.7) becomes important if the average size of the monopole is even smaller.

7 Summary and discussion

We studied the dual gauge model of the long-distance Yang-Mills theory in terms of two-point Wightman functions. The quantization of the model has been provided by using CCR, thus avoiding other methods (e.g., path integral use). We intended to give our understanding of the confinement by making use of nothing else but the well-known tools of quantum field theory based on LD given in [12] as well as the renormalization model and symmetry properties. Among the physicists dealing with the models of interplay of a scalar (monopole, Higgs) field with a dual vector (gauge) boson field, where the vacuum state of the quantum Y-M theory is realized by a condensate of the monopole-antimonopole pairs, there is a strong belief that the flux-tube solution explains the scenarios of color confinement. Based on the flux-tube scheme approach of Abelian dominance and monopole condensation, we have obtained the analytic expressions for both the monopole and dual gauge boson field propagators (4.5) and (4.10), respectively, in $S'(\Re_4)$. These propagators lead to a consistent perturbative expansion of Green's functions. However, the Fourier transformation of the first term in TPWF (2.14) gives the occurrence of the $\delta'(p_\mu^2)$-function. This is a consequence of the nonunitarity of the translations, and the spectral function with such a term gives an indefinite metric [16]. We have found that the Goldstone theorem is valid in our model in the form taking into account the Dirac's string effect (4.12). In fact, we obtained that the characteristic $\delta(p_\mu^2)$ term naturally appears in the Fourier transformation of the v.e.v. of the commutator (4.11) of the monopole current and the scalar local field $\bar{b}$. In principle, a similar result has to be expected if one replaces the $\bar{b}$-field in (4.11) by any product of the gauge field C_μ and $\bar{b}$. We see that the fields $b(x)$ and $b_3(x)$ receive their masses and the $\bar{b}(x)$ field in combination with $\partial^\nu \tilde{G}_{\mu\nu}(x)$ form the vector field $C_\mu(x)$ obeying the equation of motion for the massive vector field with the mass $m = g\,B_0$. The solution of the $\bar{b}(x)$-field can be identified as a "ghost"-like particle in the substitute manner.

The occurrence of an arbitrary parameter l in (2.14) and (2.15) leads to breaking their covariance under the dilatation transformation (3.2) and provides spontaneous symmetry breaking of the dilatation invariance of Eq. (2.10). The monopole condensation, formulated in the framework of LDY-M model, causes the strong and long-range interplay between heavy quark and antiquark, which gives the confining force, through the dual Higgs mecha-

nism. We have obtained the analytic expression for the static potential (6.7) at large distances. The form of this potential grows linearly with the distance r apart from logarithmic correction. The latter comes from the second term in the expression (6.5) (see also (6.6)).

Making an analytic comparison of ϵ_1 (6.10) and ϵ_2 (6.11) with a in (6.9), one can conclude that we have obtained a similar behaviour of the string tension a to those in the magnetic flux picture of the vortex and in the Nambu scheme, respectively, as well as in the dual Ginzburg-Landau model [9].

Finally, it is to be noted that we have played the game with the choice of the gauge group where the Abelian group appears as a subgroup of the full Y-M gauge group. This is a very instructive method of calculating the confinement potential in the static limit in the analytic form. However, we understand that no real physics can depend on such a choice. Now, there is a next step in more formal consideration of the Y-M theory where it seems to be a new mechanism of confinement [28,29].

8 Acknowledgements

G.A.K. is grateful to G.M. Prosperi for the kind hospitality at the University of Milan where this work has partly been done.

9 Appendix

The following relations [21,16,15] were used for calculation of the coefficients a_1 and a_2 in (4.1):

$$D_2(x^0 = 0, \vec{x}) = 0 \ , \quad \partial_\mu \, D_2(x)_{|_{x^0=0}} = g_{o\mu} \, \delta_3(\vec{x}) \ ;$$

$$(\Delta^2)^2 \, E_2(x) = \partial_0^2 \, E_2(x)_{|_{x^0=0}} = \partial_0 \, E_2(x)_{|_{x^0=0}} = E_2(0, \vec{x}) = 0 \ ;$$

$$\partial_0^3 \, E_2(x)_{|_{x^0=0}} = 8 \, \pi \, g_{0\mu} \, \delta_3(\vec{x}) \ , \quad \Delta^2 \, E_2(x) = D_2(x) \ .$$

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